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The transport coefficient $\hat{q}$ in an anisotropic plasma

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We investigate the jet quenching parameter in the case of a fast moving quark in an anisotropic plasma. In the leading log approximation, strong indications are found that the transport coefficient increases with increasing anisotropy. Implications for the phenomenology at RHIC are discussed.

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1. Introduction

In this talk, which is based on the work [1], the transport coefficient in an anisotropic but homogenous plasma at high temperature is discussed. The quoted paper contains a full list of references.

The observation at RHIC of a strong suppression of large momentum particles in Au-Au collisions when compared to proton-proton collisions scaled with the number of participant nucleons, is certainly a strong evidence for the formation of a dense partonic medium, as e.g. reviewed recently in [2].

Jet quenching is usually attributed to radiative parton energy loss in the medium (for reviews, see: [3, 4, 5, 6]), where the properties of the medium are encoded in the transport coefficient $\hat{q}$ that is defined as the ratio of the mean p_t^2, transfered from the plasma to the hard partons propagating through the medium and the hard parton path length in the medium. Therefore, the jet quenching parameter is also related to the p_t-broadening of the energetic parton. More precisely, it is given by

$$\hat{q} = \rho \int d^2 q_\perp q_\perp^2 \frac{d\sigma}{d^2 q_\perp} \,, \tag{1.1}$$

where ρ is the number density of the constituents of the medium, and $\frac{d\sigma}{d^2 q_\perp}$ is the differential scattering cross section of the parton (massless quark or gluon) on the medium. Depending on the model used in phenomenological works and because of the many theoretical uncertainties, $\hat{q}$ may be quoted in wide range of $0.5 - 20\ GeV^2/fm$ see e.g [7, 8]. Therefore, the information about the nature of the medium contained in the transport coefficient is still uncertain. In these studies, so far, the medium is assumed to be isotropic.

It has been found (see for a recent review: [9, 10] and e.g. [11, 12, 13]) that the physics of anisotropic plasmas differs from that of isotropic ones, because of the presence of plasma instabilities in the former. Due to these observations it requires to reanalyse $\hat{q}$ in the context of anisotropic plasmas. Momentum broadening in a homogenous but locally anisotropic high-temperature system for a heavy quark induced by collisions has been discussed recently [14] (see also [15, 16]). To include the finite temperature dependence in the quark-medium interaction we shall consider the thermal quark self-energy [17] in the approximation where the hard quark of momentum p remains on-shell at temperature $T = 0$. Thus, Eq. (1.1) reads,

$$\hat{q} = -\frac{g^2 C_F}{2p^0}\ \mathrm{Im} \int \frac{d^4 q}{(2\pi)^4}\ q_\perp^2\ 2\pi\delta_+((p-q)^2)(1+f(q^0))\mathrm{Tr}\,[\not{p}\gamma^\mu(\not{p}-\not{q})\gamma^\nu]\Delta_{\mu\nu} \,, \tag{1.2}$$

(c.f. Fig. 1), where $f(q^0) = (\exp(q^0/T)-1)^{-1}$ is the Bose-Einstein distribution, and $\Delta_{\mu\nu}$ denotes the gluon propagator. In the eikonal approximation, when $q^0 << T$ and $q^0/p^0 \to 0$, one may simplify further Eq. (1.2) :

$$\hat{q} = -g^2 C_F\ \mathrm{Im} \int \frac{d^4 q}{(2\pi)^4}\ q_\perp^2\ \frac{2T}{q^0} 2\pi\delta(q^0 - \vec{V}\cdot\vec{q})\ \frac{p\cdot\Delta\cdot p}{(p^0)^2} \,, \tag{1.3}$$

where $\vec{V} = \vec{p}/p^0$ is the quark velocity. In what follows, we shall consider a massless quark, therefore we have $\vec{V}^2 = 1$. Concerning kinematics: $\vec{V}\cdot\vec{q} = |\vec{q}|\cos\theta_{pq} = q^0$, $q_\perp^2 = |\vec{q}|^2\sin^2\theta_{pq} = |\vec{q}|^2(1-x^2)$ with $x = q^0/|\vec{q}|$.

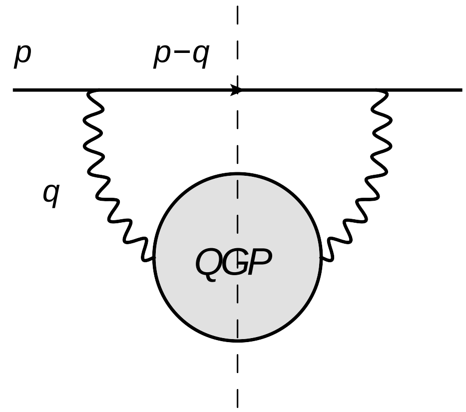

Figure 1: Diagramatic representation of Eq. (1.2).

As already pointed out in [14] this equation (1.3) tells us that all the information about the medium, either isotropic or anisotropic, is contained in the imaginary part of the gluon propagator which is evaluated in the Hard-Thermal-Loop framework.

2. $\hat{q}$ from Hard-Thermal-Loop

In the Hard-Thermal-Loop approximation [17, 18] the inverse retarded gauge-field propagator in covariant gauge reads

$$\left(\Delta^{-1}\right)^{\mu\nu} = -q^2 g^{\mu\nu} + q^\mu q^\nu - \Pi^{\mu\nu} - \frac{1}{\lambda} q^\mu q^\nu \ . \tag{2.1}$$

The gluon self-energy is given by

$$\Pi^{\mu\nu}(q) = g^2 \int \frac{d^3\vec{p}}{(2\pi)^3} v^\mu \frac{\partial n(\vec{p})}{\partial p^\beta} \left(g^{\nu\beta} - \frac{v^\nu q^\beta}{q \cdot v + i\varepsilon} \right) , \tag{2.2}$$

where $v^\mu \equiv (1, \vec{p}/|\vec{p}|)$ is a light-like vector describing the propagation of a plasma particle in space-time.

Following [11, 13], the phase space distribution function for an anisotropic plasma is taken as follows

$$n(\vec{p}) = N(\xi) n_{\rm iso}\left(\sqrt{\vec{p}^2 + \xi(\vec{p}\cdot\vec{n})^2} \right) . \tag{2.3}$$

Thus, $n(\vec{p})$ is obtained from an isotropic distribution $n_{\rm iso}(|\vec{p}|)$ by removing particles with a large momentum component along $\vec{n}$. $N(\xi) = \sqrt{1+\xi}$ is a normalization factor which insures that $\int d^3p\; n(\vec{p}) = \int d^3p\; n_{\rm iso}(\vec{p})$. In what follows, we shall omit this factor. Its effects on $\hat{q}$ will be discussed at the end of the section.
For evaluating Eq. (1.3) we choose the reference frame in which the initial energetic (hard) quark propagates along the z-axis, i.e, $\vec{V} = (0,0,1)$, whereas the beam nuclei collide along the y-axis,

which is the direction of anisotropy, denoted by the three-dimensional unit vector $\mathbf{n} = (0,1,0)$, namely, $\vec{V} \perp \vec{n}$ which refered to a quark produced at mid-rapidity .

For an anisotropic plasma, the gluon propagator reads

$$\hat{p}\cdot\Delta\cdot\hat{p} = \Delta_A \left[1 - x^2 - \frac{x^2\hat{q}_y^2}{1-\hat{q}_y^2}\right] + \Delta_G \left[x^2(q^2-\alpha-\gamma) + \frac{x^2\hat{q}_y^2}{1-\hat{q}_y^2}(\omega^2-\beta) - 2x^2\hat{q}_y\delta\right] , \tag{2.4}$$

where

$$\Delta_A^{-1} = (q^2-\alpha) , \tag{2.5}$$

and

$$\Delta_G^{-1} = (q^2-\alpha-\gamma)(\omega^2-\beta) + \delta^2\vec{q}^{\,2}\tilde{n}^2 , \tag{2.6}$$

with the transverse momentum component $\hat{q}_y = \vec{q}\cdot\vec{n}/|\vec{q}| = q_y/|\vec{q}|$ into the direction of anisotropy. $\omega = q\cdot u$ where u^μ is the heat-bath vector, which in the local rest frame is given by $u^\mu = (1,0,0,0)$ The functions α, β, γ and δ, are obtained from Eq. (2.2). Explicit expressions maybe found e.g. in [11, 14].

The contribution of Δ_A to $\hat{q}$ of Eq. (1.3) is considered, which even to LL accuracy shows the possible presence of the plasma instability. In performing the $|\vec{q}| = q-$integration, the contribution at LL accuracy reads

$$\hat{q}_A = -\frac{g^2C_FT}{4\pi^3}\int d\Omega_q \frac{1-x^2}{x}\left[1-x^2-\frac{x^2\hat{q}_y^2}{1-\hat{q}_y^2}\right] I(x,\alpha) , \tag{2.7}$$

with

$$I(x,\alpha) \simeq \frac{\mathrm{Im}\alpha}{(1-x^2)^2}\left[\frac{1}{2}\ln\frac{T}{m_D} + \frac{\pi}{2}\frac{\mathrm{Re}\alpha}{\mathrm{Im}\alpha}\Theta(-\mathrm{Re}\alpha)\right] , \tag{2.8}$$

where

$$\mathrm{Im}\alpha \simeq -\frac{\pi}{4}\,x(1-x^2)m_D^2\left\{1+\frac{\xi}{2}[3\hat{q}_y^2-1-x^2(5\hat{q}_y^2-1)]\right\} , \tag{2.9}$$

and

$$\mathrm{Re}\alpha \simeq -\frac{1}{3}\xi\hat{q}_y^2 m_D^2 , \tag{2.10}$$

with m_D the isotropic Debye mass. The second term in Eq. (2.8) has to be kept, because it reflects a singularity for $\mathrm{Im}\alpha \propto x \to 0$ due to the anisotropy $\xi > 0$ that reflects the plasma instabilities present in an anisotropic plasma.

At small anisotropy, i.e., $\xi \ll 1$, analytic calculations are possible. Following [14] the contribution from the first term in Eq. (2.8) to $\hat{q}_A$ is denoted as regular. After performing the angular integrations in Eq. (2.7) it leads at LL order with $T >> m_D$ to

$$\hat{q}_A^{reg} = \frac{g^2C_Fm_D^2T}{8\pi}\ln\frac{T}{m_D}\,(1+O(\xi^2)) , \tag{2.11}$$

with no contribution at first order in ξ. Applying the same procedure for the second term Δ_G we obtain

$$\hat{q}_G^{reg} = \frac{3g^2 C_F m_D^2 T}{8\pi} \ \ln \frac{T}{m_D} \ (1 + O(\xi^2)). \tag{2.12}$$

with no singular contribution at this order.

Summing the two terms Eqs. (2.11) and (2.12) the LL transport coefficient in the limit of small ξ up to $O(\xi)$ becomes

$$\hat{q}_{anisio}^{reg} = \hat{q}_A^{reg} + \hat{q}_G^{reg} = \frac{g^2 C_F m_D^2 T}{2\pi} \ \ln \frac{T}{m_D} \ (1 + O(\xi^2)), \tag{2.13}$$

which reproduces $\hat{q}$ in an isotropic plasma ($\xi = 0$).

Next the anomalous contribution [14] due to the second term of Eq. (2.8) is evaluated. In LL order only the behavior for $x \to 0$ is relevant. With Eq. (2.10) it gives

$$\hat{q}_A^{anom} \simeq \frac{g^2 C_F m_D^2 T}{24\pi^2} \xi \ \int d\Omega_q \frac{\hat{q}_y^2}{x} \ , \tag{2.14}$$

inducing a logarithmic singularity with $x = \cos\theta_{pq}$. The contribution to $\hat{q}_G^{anom}$ starts at $O(\xi^2)$. We note that the anomalous part (2.14), that is the first correction produced by the anisotropy is positive. Therefore, the transport coefficient is enhanced by the anisotropy.

To cut the singularity we suggest three possibilities:

(i) One may follow the detailed and plausible arguments given in [14] that this soft singularity is screened by $O(g^3)$ terms in the gluon propagator, i.e. beyond the HTL approximation under discussion. It leads to the replacement of Imα in the second term in the denominator of Eq. (2.8) by Im$\alpha \sim x \to x + cg$, i.e. it is suggestive to cut the singularity in (2.14) by

$$\xi \int \frac{dx}{x} \to 2\xi \int_0 \frac{dx}{x + cg} \sim 2\xi \ln \frac{1}{g} \sim 2\xi \ln \frac{T}{m_D} \ . \tag{2.15}$$

This way a finite result is obtained,

$$\hat{q}_A^{anom} \simeq \frac{g^2 C_F m_D^2 T}{2\pi} \ln \frac{T}{m_D} \ \frac{\xi}{6} \ , \tag{2.16}$$

which shows a positive, but weak dependence on ξ as a sign of the anisotropy for $\xi > 0$..

(ii) The origin of the $1/x$ singularity is traced back to the Bose-Einstein distribution $f(q^0) \sim T/q^0$ in Eq. (1.2). Pragmatically, in the anisotropic case, this behavior could be modified by $q^0 \to \sqrt{(q^0)^2 + \xi(\vec{n} \cdot \vec{q})^2} = q \ \sqrt{x^2 + \xi \hat{q}_y^2}$, $q^0 > 0$. On mass-shell this replacement gives the distribution in Eq. (2.3), and leads to

$$\xi \int \frac{dx}{x} \to 2\xi \ln \frac{1}{\xi} \ . \tag{2.17}$$

(iii) To form the anisotropic configuration in momentum space a characteristic time scale is present of the order $\tau_c \sim O(1/g\xi T)$, for not to large ξ. It is then natural to cut the energies of the constituents in the heat bath by $|q^0| \geq 1/\tau_c > g\xi T$, and

$$\xi \int \frac{dx}{x} \to 2\xi \ln \frac{1}{g\xi} \ . \tag{2.18}$$

In summary all three options Eqs. (2.15 - 2.18) lead to a positive contribution of $O(\xi)$ at LL order to $\hat{q}_{aniso}$. Also, if we take into account the normalization factor $N(\xi)$ one gets an additional enhancement of $\hat{q}$ by a factor $\sqrt{1+\xi} \simeq 1 + \frac{\xi}{2}$ at small ξ.

3. Conclusion

To summarize, the transport coefficient for an anisotropic plasma is shown to be larger than that in an isotropic one, at least at small anisotropy for which the calculations are performed. However, a detailed quantitative study, for larger anisotropy, is needed to get an overall estimate of anisotropy effects on the jet-quenching parameter and for possible phenomenological applications at RHIC and LHC. Indeed, a better handling of the theoretical value of the transport coefficient can be essential to distinguish the various phenomenological models. Note that similar conclusions have been reached independently in [16, 19, 20, 21].

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